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Technical Memorandum

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ABSTRACT

A new model for the elevation spectrum of wind-generated ocean waves has been developed. The proposed model is a modification of the Pierson model spectrum (Ref. 1). An error in the Pierson model has been corrected, the lower bound of the Mitsuyasu-Honda spectral range has been increased, and the Leykin-Rosenberg spectral range has been eliminated. The result of these modifications is a simpler model that still compares favorably with existing data.

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SUMMARY

A new model for the elevation spectrum of wind-generated ocean waves has been developed. It is a modification of the Pierson model spectrum (Ref. 1), which is a revision of the earlier Pierson-Stacy model spectrum (Ref. 2). The model proposed herein corrects an error discovered in the model described in Ref. 1. While investigating this error, other modifications and possible simplifications were noted and have been incorporated into the proposed model. The wave number lower bound of the Mitsuyasu-Honda range has been increased to agree better with experimental observation, and one of the five ranges in the Pierson model has been eliminated. In the present report, we discuss the form of the proposed model spectrum, develop the motives for the modifications to the Pierson model, and compare the proposed model with experimental measurements.

MODEL EQUATIONS

The air-sea interface separating the world's atmosphere from its oceans is a complicated two-dimensional surface that can be described by a vertical elevation coordinate and two perpendicular horizontal coordinates. It is often helpful to decompose the surface elevation into its Fourier components in the two horizontal directions. The squared magnitude of this two-dimensional Fourier transform is called the power spectral density of the sea surface elevation or simply the sea surface elevation spectrum. This elevation spectrum is often used to describe the sea surface in theoretical predictions. To simplify calculations involving the sea surface elevation spectrum, various models have been used, each described by only a few parameters. The kinds of problems in which a model of the sea surface elevation spectrum plays a role include the prediction of sun glitter from the ocean, the interaction between ocean surface waves and internal waves or surface currents, and the prediction of radar backscatter from the ocean surface.

The sea surface elevation spectrum as a function of wave vector $\vec{k}$ may be written as

$$\hat{S}(\vec{k}) = \hat{S}(k,\phi) = S(k)F(k,\phi) \tag{1}$$

where $k = |\vec{k}|$ and ϕ is measured relative to the average direction of wave propagation. Equation 1 is defined for $0 < k < \infty$ and $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$; it is equal to zero elsewhere. The model proposed herein consists of analytical forms for $S(k)$, often called the direction-independent part of the sea surface elevation spectrum. The model is expected to be reliable for winds described by a friction velocity $u_* \geq 12$ cm/s. The form for $F(k,\phi)$ may be found in Ref. 2.

Two representations of the model are given in this section. The first is the sea surface elevation spectrum as a function of wave number, $S(k)$. The second is the sea surface elevation spectrum as a function of temporal radian frequency, $S(\omega)$. The relation between $S(\omega)$ and $S(k)$ is given by

$$S(k) = S[\omega(k)] \frac{d\omega(k)}{dk} \tag{2}$$

where, in general, $\omega^2 = gk + \frac{\tau k^3}{\rho} = gk\left(1 + \frac{k^2}{k_m^2}\right)$, (3)

with $k_m^2 = \frac{g\rho}{\tau}$. In these relations, g is the acceleration of gravity, ρ is the sea water density, and τ is the surface tension. For the gravity range $k << k_m$, this dispersion relation can be approximated as

$$\omega^2 \cong gk. \tag{4}$$

Similarly, for the capillary range $k >> k_m$, the relation can be approximated as

$$\omega^2 \cong \frac{gk^3}{k_m^2} = \frac{\tau}{\rho} k^3 . \tag{5}$$

EQUATIONS FOR S(k)

$$S(k) = \begin{cases} S_1(k) & 0 < k < k_0 & \text{Pierson-Moskowitz range} \\ S_1^*(k) & k_0 < k < k_1 & \text{Stacy range} \\ S_2^*(k) & k_1 < k < k_2 & \text{Kitaigorodskii range} \\ S_3(k) & k_2 < k < k_\nu & \text{Mitsuyasu-Honda range} \\ S_4(k) & k_\nu < k < \infty & \text{Cox viscous cutoff range} \end{cases}$$

$$S_1(k) = \frac{\alpha}{2k^3} \exp(-Bk^{-2}) \text{ for } 0 < k < k_0 \tag{6}$$

$$S_1^*(k) = \max\begin{cases} S_1(k) \\ S_s(k) \end{cases} \quad \text{for } k_0 < k < k_1 \text{ and } u_* \geq 35.8$$

$$S_1^*(k) = S_1(k) \quad \text{if} \quad u_* < 35.8$$

$$S_s(k) = (271.5 + 13.58u_*) \sqrt{\frac{g}{k}} \exp\left[2.53\ (0.4\pi - \sqrt{gk})\right] \tag{7}$$

$$S_2{*}(k) = \max \begin{cases} S_s(k) \\ S_2(k) \end{cases} \qquad \text{for } k_1 < k < k_2 \text{ and } u_* \quad 75.76.$$

$$S_2{*}(k) = S_2(k) \qquad \text{if } u_* < 75.76.$$

$$S_2(k) = 0.4375 \left(\frac{2\pi}{\sqrt{g}}\right)^{p-1} \frac{1 + \frac{3k^2}{k_m^2}}{\left[k_2\left(1 + \frac{k_2^2}{k_m^2}\right)\right]^{\frac{p-4}{2}} \left[k\left(1 + \frac{k^2}{k_m^2}\right)\right]^{5/2}} \tag{8}$$

$$p = 5 - \log_{10}(u_*).$$

$$S_3(k) = 0.4375 \frac{(2\pi)^{p-1}\left(1 + \frac{3k^2}{k_m^2}\right)}{g^{\frac{p-1}{2}}\left[k\left(1 + \frac{k^2}{k_m^2}\right)\right]^{\frac{p+1}{2}}} \quad \text{for } k_2 < k < k_\nu. \tag{9}$$

$$S_4(k) = 1.473 \times 10^{-4} \frac{u_*^3 k_m^6}{k^9} \quad \text{for } k < k_\nu. \tag{10}$$

Parameters:

$$k_0 = \sqrt{\frac{2\beta}{3}} \; \frac{g}{U_{19.5}^2}$$

$$k_1 = \left(\frac{\alpha}{0.875}\right)^2 \left(\frac{g}{4\pi^2}\right)^{p-1} \left[k_2\left(1 + \frac{k_2^{\,2}}{k_m^{\,2}}\right)\right]^{p-4} \frac{\left(1 + \frac{k_1^{\,2}}{k_m^{\,2}}\right)^5}{\left(1 + \frac{3k_1^{\,2}}{k_m^{\,2}}\right)^2}$$

(The solution of this equation for k_1 is discussed in the following section, "Modifications Incorporated into New Model," where it appears as Eq. 25.)

$k_2 = k(20\pi) = 2.639$ rad/cm

k_ν = point where $S_3(k)$ intersects $S_4(k)$

$U_{19.5}$ = wind velocity at 19.5 m in cm/s (See the discussion in the following section under "Wind Profile Relationship.")

u_* = wind friction velocity in cm/s

$\alpha = 8.1 \times 10^{-3}$

$\beta = 0.74$

$B = \beta g^2/U_{19.5}^{\,4}$

$k_m = \sqrt{g\rho/\tau} = 3.63$ rad/cm

$g = 981$ cm/s^2

$\rho = 1.025$ g/cm^3

$\tau = 76.3$ dyne/cm

$u_{*m} = 12.0$ cm/s

EQUATIONS FOR S(ω)

$$S(\omega) = \begin{cases} S_1(\omega) & 0 < \omega < \omega_0 & \text{Pierson-Moskowitz range} \\ S_1^*(\omega) & \omega_0 < \omega < \omega_1 & \text{Stacy range} \\ S_2^*(\omega) & \omega_1 < \omega < \omega_2 & \text{Kitaigorodskii range} \\ S_3(\omega) & \omega_2 < \omega < \omega_\nu & \text{Mitsuyasu-Honda range} \\ S_4(\omega) & \omega_\nu < \omega < \infty & \text{Cox viscous cutoff range} \end{cases}$$

$$S_1(\omega) = \frac{\alpha g^2}{\omega^5} \exp(-C\omega^{-4}) \quad \text{for } 0 < \omega < \omega_0. \tag{11}$$

$$S_1^*(\omega) = \max \begin{cases} S_1(\omega) \\ S_s(\omega) \end{cases} \quad \text{for } \omega_0 < \omega < \omega_1 \text{ and } u_* \geq 35.8.$$

$$S_1^*(\omega) = S_1(\omega) \quad \text{if} \quad u_* < 35.8.$$

$$S_s(\omega) = (543 + 27.17\, u_*) \exp[2.53\,(0.4\pi - \omega)]. \tag{12}$$

$$S_2^*(\omega) = \max \begin{cases} S_s(\omega) \\ S_2(\omega) \end{cases} \quad \text{for } \omega_1 < \omega < \omega_2 \text{ and } u_* \geq 75.76.$$

$$S_2^*(\omega) = S_2(\omega) \quad \text{if } u_* < 75.76.$$

$$S_2(\omega) = \frac{0.875\ (2\pi)^{p-1}}{\omega_2^{p-4}\ \omega^4} . \quad (13)$$

$$p = 5 - \log_{10}(u_*) .$$

$$S_3(\omega) = \frac{0.875}{2\pi} \left(\frac{2\pi}{\omega}\right)^p \quad \text{for } \omega_2 < \omega < \omega_\nu . \quad (14)$$

$$S_4(\omega) = \frac{0.982 \times 10^{-4}}{\omega^{19/3}} u_*^3\ g^{8/3}\ k_m^{2/3} \quad \text{for } \omega > \omega_\nu . \quad (15)$$

Parameters:

$$\omega_0 = \sqrt[4]{\frac{4\beta}{5}}\ \frac{g}{U_{19.5}}$$

$$\omega_1 = \left(gk_1 + \frac{\tau}{\rho} k_1^3\right)^{1/2} = \left[gk_1 \left(1 + \frac{k_1^2}{k_m^2}\right)\right]^{1/2}$$

$$\omega_2 = 20\pi$$

ω_ν = point where $S_3(\omega)$ intersects $S_4(\omega)$

$$C = \frac{\beta g^4}{U_{19.5}^4}$$

MODIFICATIONS INCORPORATED INTO NEW MODEL

The model spectrum proposed in the previous section includes three modifications to the Pierson model spectrum described in Ref. 1. Each of these will be described below together with its rationale.

CORRECTION TO MITSUYASU-HONDA RANGE

The spectral form for the Mitsuyasu-Honda range given in Eqs. 6.7 and 6.13 of Ref. 1 is in error. The expression should be multiplied by a factor of 1/2. The error has been acknowledged by Professor Pierson, and the correction has been confirmed. The discovery of this error actually triggered the following modifications.

CHANGE IN THE LOWER BOUND OF THE MITSUYASU-HONDA RANGE

According to Ref. 2 (pp. 54-67), the sea surface elevation spectrum, $S(k)$, should be proportional to the friction velocity, u_*, in the Kitaigorodskii range. Furthermore, an inspection of Fig. 8.2 in the same reference reveals that $S(k) \propto u_*^b$ with $b > 1$ for the range of wave numbers between the Kitaigorodskii range and the Cox viscous cutoff range. In Ref. 3, Mitsuyasu describes good agreement between his data and a model spectrum with the property of $S(\omega) \propto u_*$ for $12\pi < \omega < 28\pi$, a range of radian frequency corresponding roughly to the Mitsuyasu-Honda range. Thus, any proposed spectral model should have a wind-speed dependence given by $S(k) \propto u_*^b$, with $b > 1$, for wave numbers between the Pierson-Moskowitz range and the Cox viscous cutoff range.

In order to ensure that $b > 1$ in the Mitsuyasu-Honda range, the lower bound of that range must be no smaller than $k(20\pi) = 2.639$ rad/cm. This may be demonstrated as follows. The spectral form used for the Mitsuyasu-Honda range is Eq. 9, rewritten here as

$$S_3(k) = 0.4375\ (2\pi)^{p-1} \frac{g\left(1 + \frac{3k^2}{k_m^2}\right)}{\left[gk\left(1 + \frac{k^2}{k_m^2}\right)\right]^{\frac{p+1}{2}}} \tag{16}$$

where $p = 5 - \log_{10}u_*$.

Equation 16 can be rearranged to obtain

$$S_3(k) = 0.4375 \frac{(2\pi)^4 g\left(1 + \frac{3k^2}{k_m^2}\right)}{\left(gk + \frac{gk^3}{k_m^2}\right)^3} \left(\frac{\sqrt{gk + \frac{gk^3}{k_m^2}}}{2\pi}\right)^{\log_{10}u_*} , \tag{17}$$

which, in turn, can be rewritten as

$$S_3(k) = A \cdot B^{\log_{10}u_*} . \tag{18}$$

In anticipation of obtaining

$$S_3(k) = au_*^{\,b} \tag{19}$$

we write

$$\log_{10}S_3(k) = \log_{10}A + \log_{10}u_* \cdot \log_{10}B$$

$$= \log_{10}a + b \cdot \log_{10}u_* .$$

Comparing terms we see that

$$b = \log_{10}B , \tag{20}$$

so indeed $S_3(k)$ can be described as having the power-law dependence on u_* shown in Eq. 19.

Table 1 shows the exponent b as a function of wave number k. From the table it is clear that the exponent b will be greater than 1 only for wave numbers greater than $k(20\pi) = 2.639$ rad/cm. We propose that $k(2\pi) = 2.639$ rad/cm be used as the lower bound for the Mitsuyasu-Honda range.

Table 1

Power-law dependence of S_3.

k (rad/cm)	b
1	0.7133
2	0.9055
2.639	1.0000
3	1.0490
6	1.3725

This modification in the lower bound for the Mitsuyasu-Honda range can also be justified by the range of frequency for which Eq. 14 fits the spectra measured by Mitsuyasu and Honda. Reference 1 (p. 301) states that the results of Mitsuyasu and Honda could be fit well by the spectral form of Eq. 14 over the frequency range from 10 to 30 cyc/s. This lower bound corresponds to 20 π rad/s or, using the dispersion relation of Eq. 3, $k(20\pi) = 2.639$ rad/cm, the same result obtained from considerations of windspeed dependence.

EXTENSION OF THE KITAIGORODSKII RANGE TO THE MITSUYASU-HONDA RANGE

Wind-generated waves on the ocean surface propagate because of two restoring forces. The restoring force of the longer waves results primarily from the earth's gravity, while the restoring force of the shorter waves results primarily from the surface tension of

the water. Long waves having frequencies of less than 1 cyc/s are called gravity waves, while short waves having frequencies greater than 10 cyc/s are called capillary waves. The gravity waves are well described by the Pierson-Moskowitz and Stacy regions of the model spectrum, while the Mitsuyasu-Honda region describes the capillary waves. However, some intermediate form is required in the transition region where neither gravity forces nor surface tension forces dominate.

The Pierson model uses two intermediate spectral forms: (a) the Kitaigorodskii region, which was established on physical grounds and joins the Stacy region, and (b) the Leykin-Rosenberg region, which was defined as a straight line on a log-log plot of power spectral density versus wave number joining the Kitaigorodskii and the Mitsuyasu-Honda regions.

The need for two intermediate spectral forms between the Stacy and the Mitsuyasu-Honda ranges was reexamined. At least one intermediate form is required because, as shown above, the form for the Mitsuyasu-Honda range should not be used for $k < 2.639$ rad/cm because of its windspeed dependence and the Stacy form will not intersect the Mitsuyasu-Honda form at $k = 2.639$ rad/cm. However, justification for a second intermediate form is less clear. It is proposed that only the Kitaigorodskii form be used in the region between the Stacy and the Mitsuyasu-Honda ranges. The Kitaigorodskii form was chosen as the transition form because the existence of such a form can be argued on physical grounds (Refs. 2 and 3). The Leykin-Rosenberg form was only a mathematical construction that could be eliminated from the proposed model.

The spectral form in the Kitaigorodskii range is given in Eq. 7.5 of Ref. 1 as

$$S_2(\omega) = \frac{Ggu_*}{\omega^4} \quad . \tag{21}$$

Because $\omega_2 = 20\pi$ rad/s ($k_2 = 2.639$ rad/cm) is in the gravity-capillary region of the spectrum, the complete dispersion relation given in Eq. 3 must be used to transform $S_2(\omega)$ into $S_2(k)$.

Following the transformation

$$S_2(k) = S_2[\omega(k)] \frac{d\omega(k)}{dk} \quad ,$$

we have

$$S_2(k) = \frac{Gu_* \left(1 + \frac{3k^2}{k_m^2}\right)}{2\sqrt{g} \left[k\left(1 + \frac{k^2}{k_m^2}\right)\right]^{5/2}} \quad , \tag{22}$$

where $k_m^2 = g\rho/\tau$.

Setting $S_2(k_2) = S_3(k_2)$, we obtain the following solution for the coefficient G:

$$G = \frac{0.875 \ (2\pi)^{p-1}}{u_* g^{\frac{p-2}{2}} \left[k_2\left(1 + \frac{k_2^2}{k_m^2}\right)\right]^{\frac{p-4}{2}}} \quad . \tag{23}$$

The spectral form $S_2(k)$ can then be written as Eq. 8:

$$S_2(k) = 0.4375\left(\frac{2\pi}{\sqrt{g}}\right)^{p-1} \frac{1 + \frac{3k^2}{k_m^2}}{\left[k_2\left(1 + \frac{k_2^2}{k_m^2}\right)\right]^{\frac{p-4}{2}} \left[k\left(1 + \frac{k^2}{k_m^2}\right)\right]^{5/2}} \tag{8}$$

where $p = 5 - \log_{10} u_*$, $k_m = 3.63$ rad/cm, and $g = 981$ cm/s^2.

The value for $k_1(u_*)$ can be determined from the relation $S_1(k_1) = S_2(k_1)$ where $S_1(k) = \frac{\alpha}{2k^3}$ for the gravity equilibrium range. From this calculation we obtain

$$k_1 = \left(\frac{\alpha}{0.875}\right)^2 \left(\frac{g}{4\pi^2}\right)^{p-1} \left[k_2 \left(1 + \frac{k_2^2}{k_m^2}\right)\right]^{p-4} \frac{\left(1 + \frac{k_1^2}{k_m^2}\right)^5}{\left(1 + \frac{3k_1^2}{k_m^2}\right)^2} , \quad (24)$$

where $\alpha = 8.1 \times 10^{-3}$.

This equation can be solved for k_1 using a simple fixed-point iteration scheme (Ref. 4). For the range of parameters of interest, the method is stable and converges quite rapidly. A few calculations yielded the results shown in Table 2 below.

Table 2

Lower bound of the Kitaigorodskii range.

u_* (cm/s)	k_1 (rad/cm)	ω_1 (rad/s)
12	0.873	30.08
24	0.226	14.91
48	0.0568	7.46
96	0.0142	3.73
192	0.00355	1.87

WIND PROFILE RELATIONSHIP

The spectrum is, of course, dependent on wind velocity. This dependence is contained in the parameters u_*, the wind friction velocity, and $U_{19.5}$, the wind velocity at a height of 19.5 m. A particular relationship between $U_{19.5}$ and u_* was given in Ref. 2. For completeness these equations are repeated here:

$$U(z) = \frac{u_*}{\kappa} \ln \frac{z}{z_0(u_*)} \tag{25}$$

where z = height above sea level expressed in cm,

$$z_0(u_*) = \frac{0.684}{u_*} + 4.28 \times 10^{-5} u_*^{2} - 0.0443, \text{ and} \tag{26}$$

κ = von Karmen's constant = 0.4.

Thus the form for $U_{19.5}$ is

$$U_{19.5} = \frac{u_*}{\kappa} \ln \frac{1950}{z_0(u_*)} \quad . \tag{27}$$

COMPARISONS WITH EXPERIMENT

Figures 1 through 5 are plots of the proposed model spectrum for various wind speeds. The plots were designed to be easily compared with the plots of the previous model given in Ref. 1.

Several comparisons were made between the proposed model and existing data. Because the Pierson-Moskowitz range and the Stacy range were compared with data in Ref. 1 and elsewhere, these comparisons will not be repeated here. The model proposed herein differs from previous models mainly in the higher frequency regions of the spectrum so these regions will be compared with existing data.

Figure 6 shows the proposed model compared to the data obtained by Kinsman (Ref. 5) and analyzed by Pierson and Stacy (Ref. 2). The windspeed dependence of the data appears to be stronger than that of the model. However, the model is consistent with the 90% confidence intervals noted on the figure. It should be noted that both the Pierson-Stacy (Ref. 2, Fig. 8.7) and the Pierson (Ref. 1, Fig. 6.9) model spectra exhibit weak windspeed dependence at this range of frequencies.

The proposed model is compared with Mitsuyasu's recently obtained data (Ref. 3) in Fig. 7. Note that the model appears to fit the data very well even though the transition to the Kitaigorodskii region ($f = 1.6$ cyc/s) does not fall exactly at the frequency ($f \cong 5$ cyc/s) where the frequency dependence of the data changes from f^{-5} to f^{-4}.

Chan and Fung (Ref. 6) compared the results of several radar backscattering coefficient measurements with the predictions of a model based in part on the Pierson model spectrum (Ref. 1). Since the proposed model has the same windspeed dependence as Pierson's model at wave numbers greater than 2.639 rad/cm, the windspeed dependence of the data cited by Chan and Fung compares as well with this proposed model as it did with the former model. However, because of the correction to the Mitsuyasu-Honda range, the proposed model results in smaller predicted backscattering coefficients than those predicted using Pierson's model, thus resulting in even stronger disagreement with the measured magnitudes of the backscattering coefficients.

The mean square slope of the surface waves can be obtained from the model by calculating the zeroth wave number moment of the

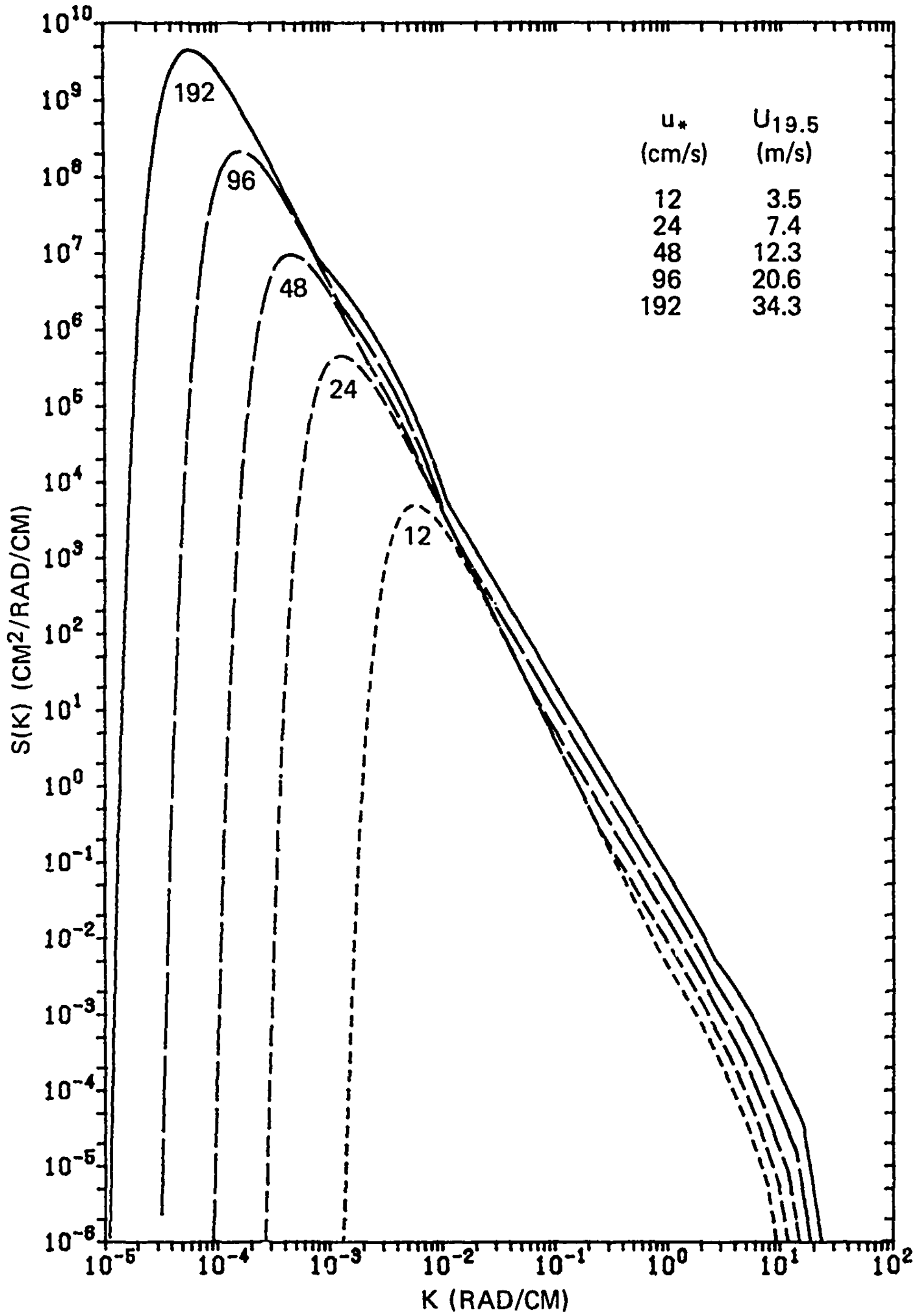

Fig. 1 Wave number elevation spectrum S(k) versus k for an extreme range of windspeeds.

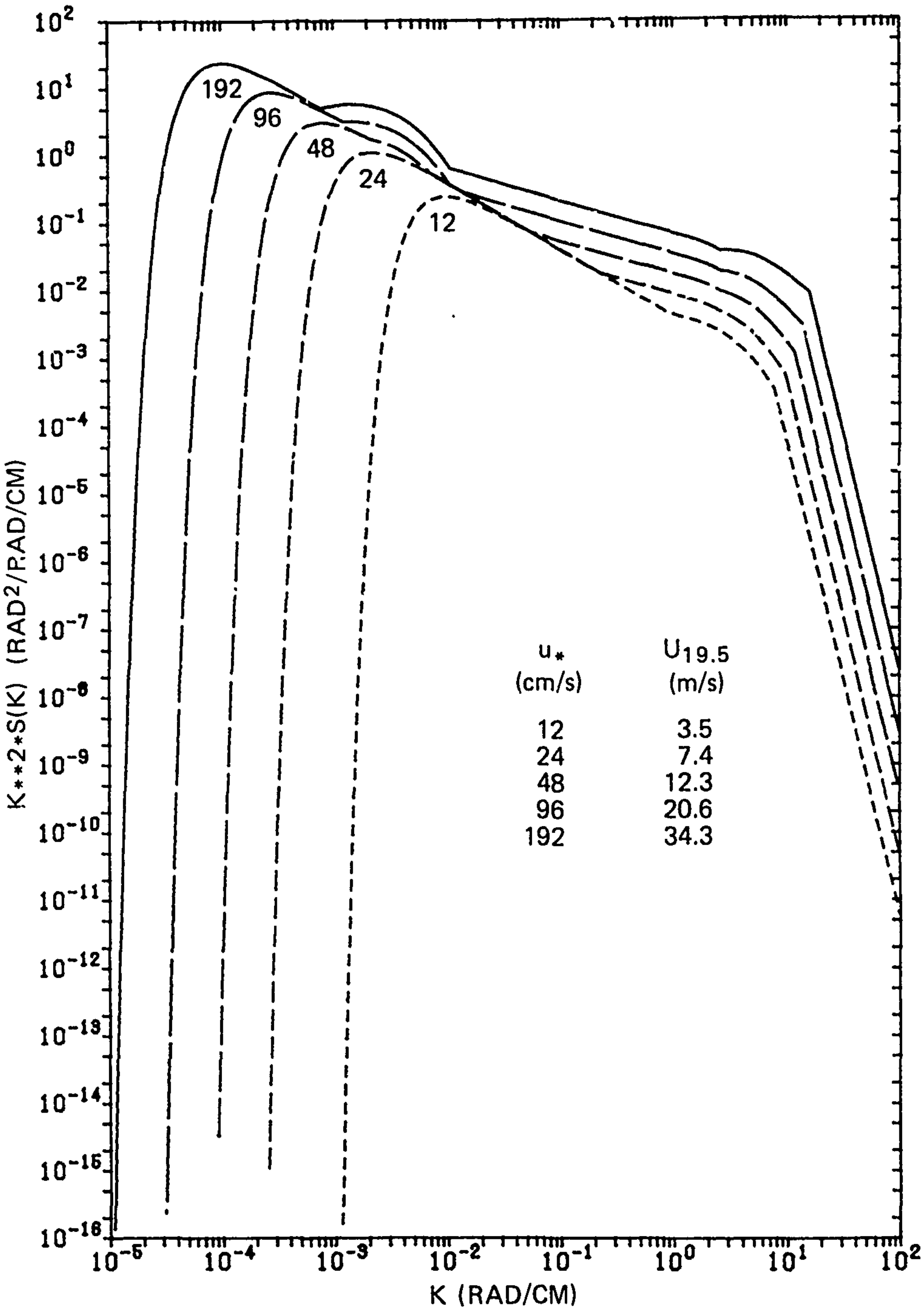

Fig. 2 **Wave number slope spectrum $k^2S(k)$ versus k for an extreme range of windspeeds.**

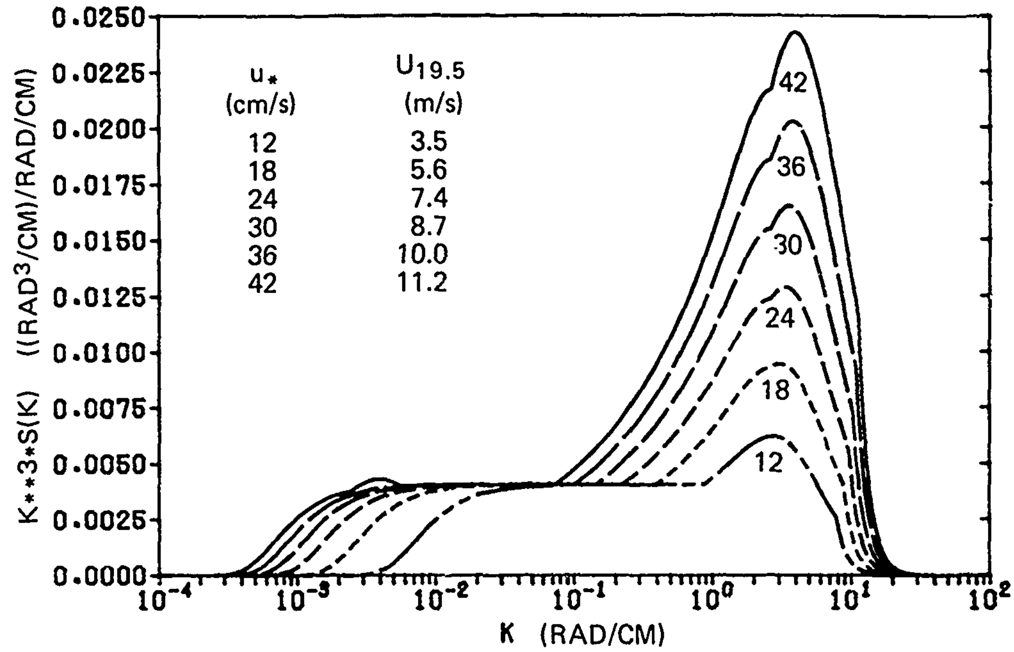

Fig. 3 $k^3S(k)$ versus k for a moderate range of windspeeds. This is a variance-preserving plot of the wave number slope spectrum (Fig. 2).

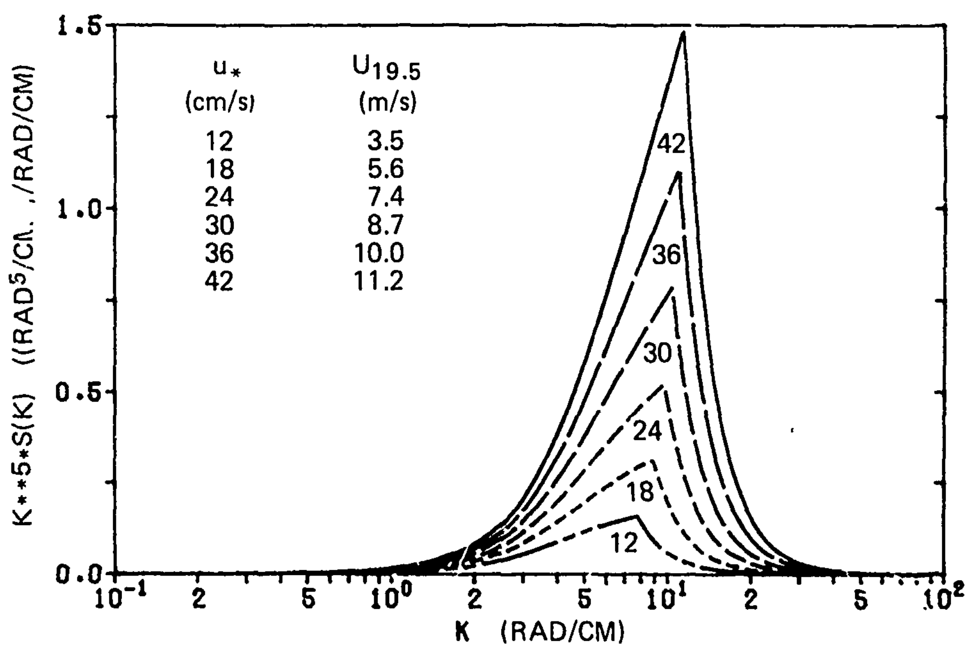

Fig. 4 $k^5S(k)$ versus k for a moderate range of windspeeds. This is a variance-preserving plot of the wave number curvature spectrum.

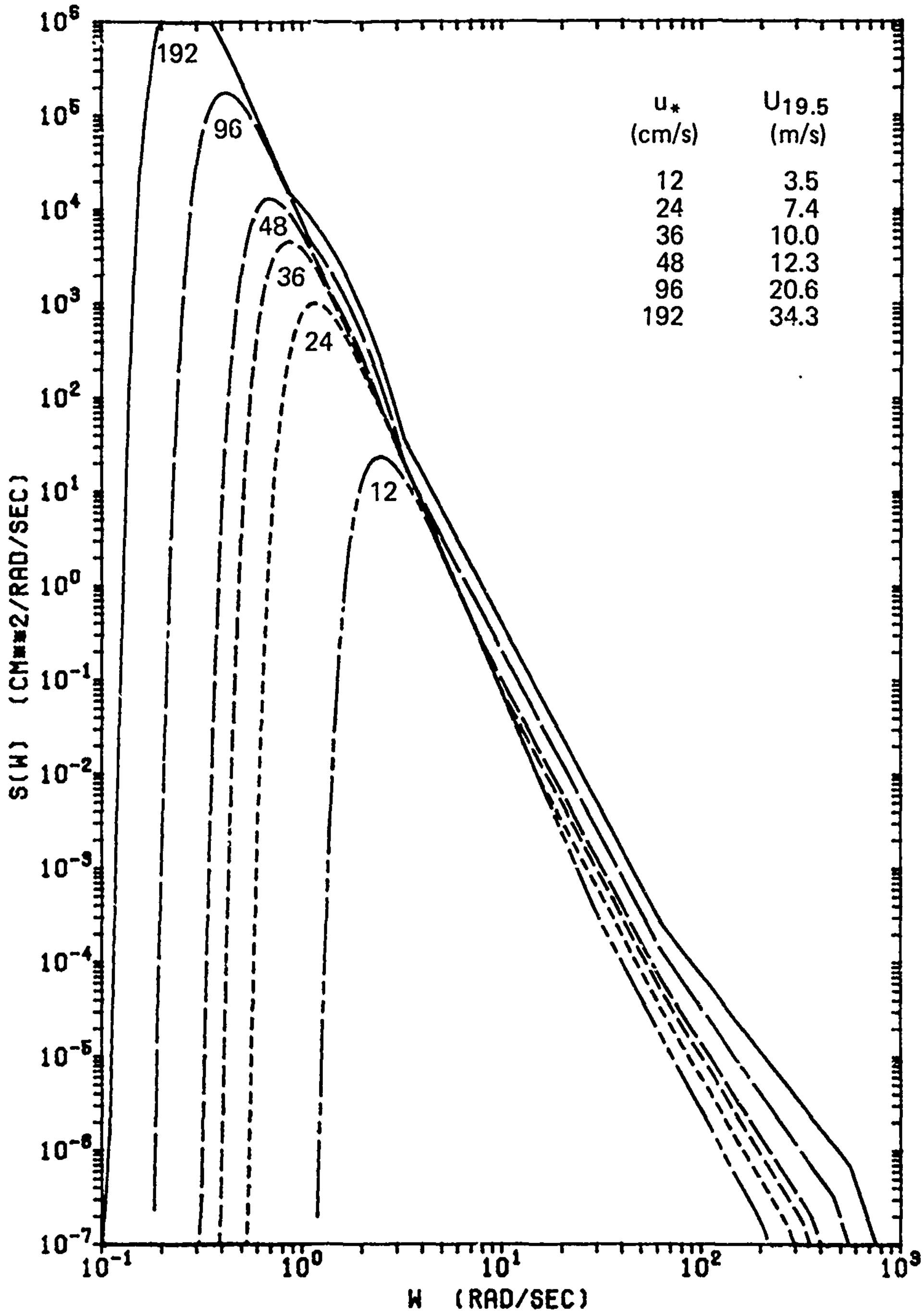

Fig. 5 Wave elevation spectrum versus radian frequency ω for an extreme range of windspeeds.

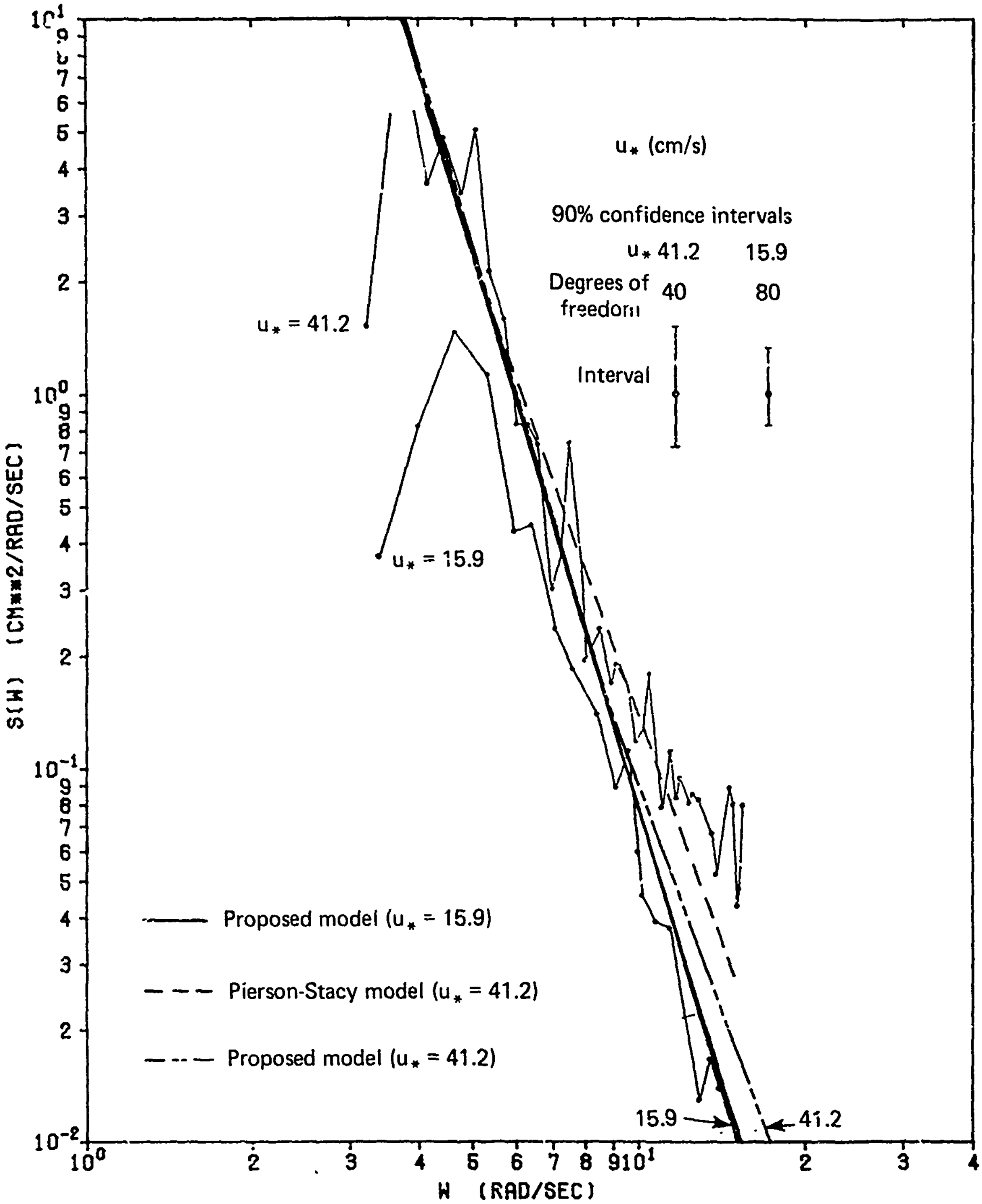

Fig. 6 Graphs of $S(\omega)$ versus ω for $u_* = 15.9$ and 41.2 cm/s. The Pierson-Stacy model and the proposed model are compared with estimated spectra determined by Pierson and Stacy (Ref. 2) from data obtained earlier by Kinsman (Ref. 5).

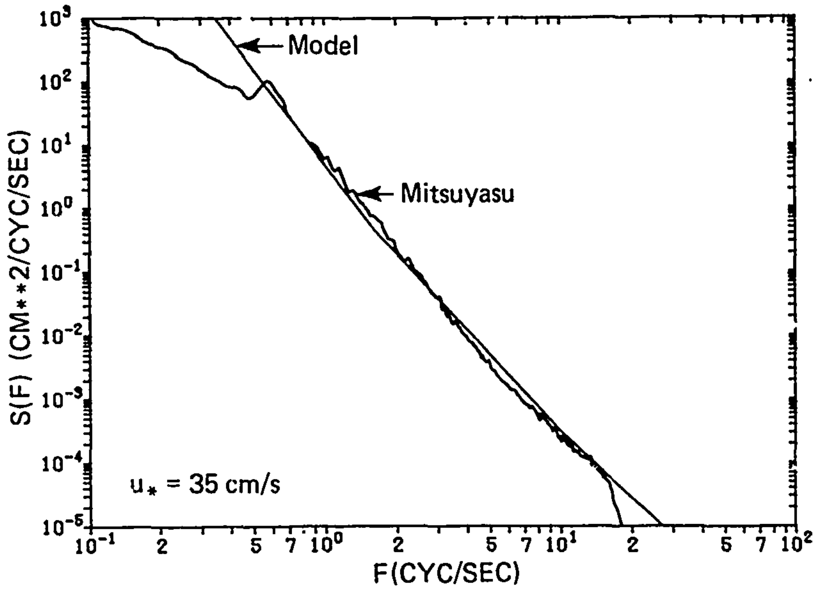

Fig. 7 Model wave elevation spectrum S(f) compared with Mitsuyasu's experimentally determined spectrum (Ref. 3).

slope spectrum or by calculating the second wave number moment of the elevation spectrum. In Fig. 8, the mean square slope thus calculated is compared with the mean square slope data of Long and Huang (Ref. 7) and Cox and Munk (Ref. 8). Notice that both models predict mean square slopes greater than those computed from measured data. However, the proposed model seems to be in closer agreement.

Various spectral moments (M_r) calculated from the relation

$$M_r = \int_0^\infty k^r \; S(k)\; dk$$

are summarized in Table 3.

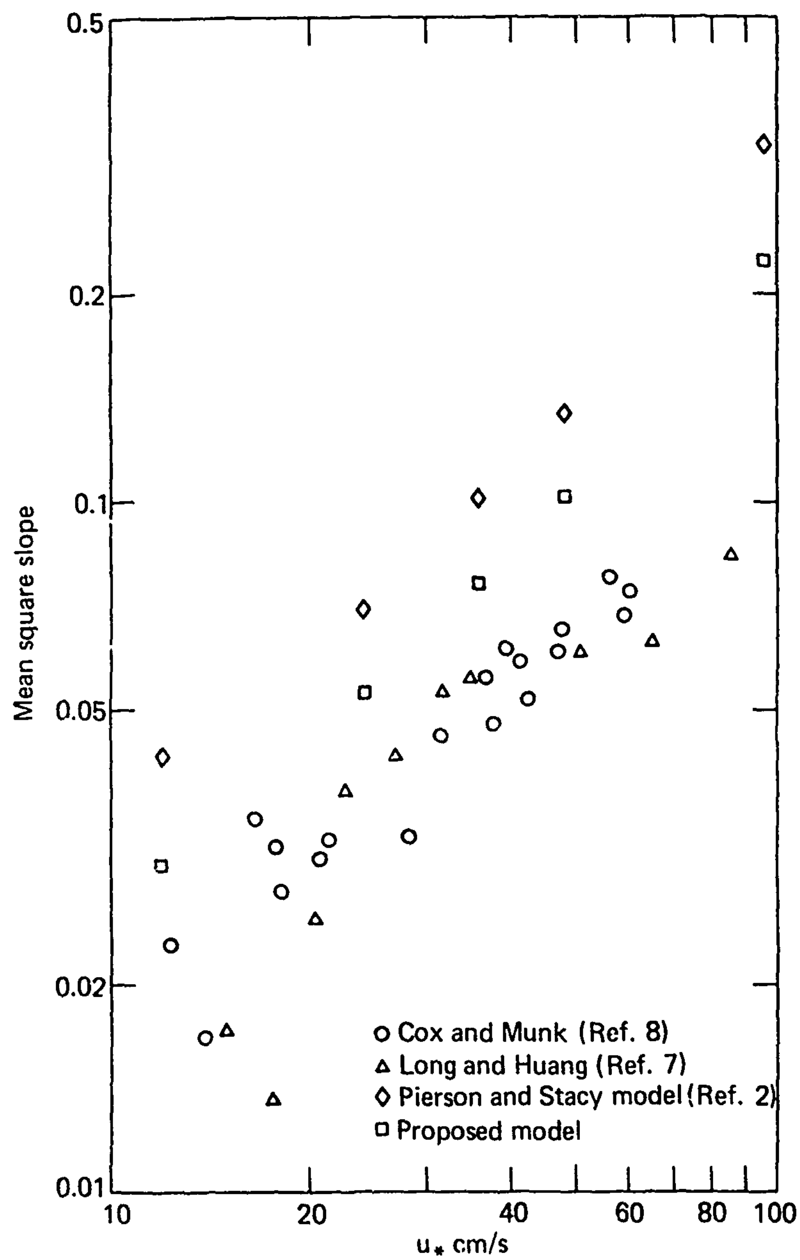

Fig. 8 Comparison of the mean square slope predicted by the proposed model with that predicted by the Pierson-Stacy model (Ref. 2) and with those measured by Long and Huang (Ref. 7) and Cox and Munk (Ref. 8).

Table 3

Spectral moments.

u_* (cm/s)	M_0 (cm^2)	M_1 (cm)	M_2 --	M_3 (cm^{-1})	M_4 (cm^{-2})
12	41.035	0.5110	0.0299	0.0404	0.1701
24	847.27	2.324	0.0531	0.1025	0.550[illegible]
36	2819.6	4.270	0.0765	0.1839	1.121
48	6587.5	5.803	0.1019	0.2828	1.897
96	52499.0	10.387	0.2224	0.8445	7.129

CONCLUSION

A new model for the direction-independent part of the ocean elevation power spectral density has been proposed. This model corrects an error found in the Pierson model (Ref. 1) and also contains several simplifications while still agreeing favorably with measured data. The overall merits of this model with only four ranges are:

1. It maintains the Pierson-Moskowitz range with the addition of the Stacy subregion of Ref. 1.
2. It shifts ω_1 to a higher frequency, making the spectrum more consistent with Mitsuyasu's results in Ref. 3.
3. It positions $k_2 = k(20\pi) = 2.639$ rad/cm so that between k_1 and k_2, $S_2(k) \propto u_*$, while above k_2 the windspeed dependence is consistent with the results of Chan and Fung in Ref. 6.

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